Effect of impurities with retarded interaction with quasiparticles upon critical temperature of s-wave superconductor.

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(Dated: May 27, 2013)

Generalization of a disordered metal's theory has been proposed when scattering of quasiparticles by impurities is caused with a retarded interaction. It was shown that in this case Anderson's theorem was violated in the sense that introduction of the impurities in s-wave superconductor increases its critical temperature. The increasing depends on parameters of the metal, impurities and their concentration. At a specific relation between the parameters the critical temperature of the dirty superconductor can essentially exceed critical temperature of pure one up to room temperature. Thus the impurities catalyze superconductivity in an originally low-temperature superconductor.

PACS numbers: 74.62.En,72.80.Ng
Keywords: dirty superconductor, impurity, Anderson's theorem, retarded interaction, catalysis

I. INTRODUCTION

Real superconductors are disordered metal containing various kinds of impurities and lattice defects. Quasi-particles scatter by these objects, that influences upon superconducting properties of a metal - critical temperature, gap, critical fields and currents change. It is well known impurities are two kinds - magnetic and nonmagnetic. The magnetic scattering differently acts on components of a cooper pair (for singlet pairing), with the result that its decay takes place. Superconducting state is unstable regard to introduction of magnetic impurities - critical temperature decreases, that is accompanied by effect of gapless superconductivity. In a case of nonmagnetic impurities an ordinary potential scattering acts on both electrons of a cooper pair equally. Therefore the pair survives. Hence, *superconducting state is stable regard to introduction of nonmagnetic impurities - a gap and critical temperature of a superconductor do not change.* This statement is Anderson's theorem - $T_{\mathtt{C}}$ and $\Delta(T)$ of an isotropic s-wave superconductor do not depend on presence of nonmagnetic impurities [1–4]. This phenomenon is result of the gap function Δ and the energetic parameter ε being renormalized equally [3]. In a case of anisotropic s-pairing a weak suppression $T_{\mathtt{C}}$ takes place by disorder [5, 6]. For d-wave pairing the nonmagnetic impurities destroy superconductivity like magnetic impurities [5–8]. Besides a superconducting state is unstable regard to introduction of nonmagnetic impurities if the gap is an odd function of $k - k_F$ [10]. If pairing of electrons with nonretarded interaction takes place then $T_{\mathtt{C}}$ quickly decreases with an increase of disorder [11].

The disorder can influence upon phonon and electron specter in materials. It results to both increase and decrease of $T_{\mathtt{C}}$. Experiments in superconducting metal showed suppression of $T_{\mathtt{C}}$ by a sufficiently strong disorder [12–15]. The strong disorder means that a free length l is such that $\frac{1}{k_F l} \approx 1$ or $\varepsilon_F \tau \approx 1$, where $\tau = l/v_F$ is a mean free time. For weak superconductors as `Al` or `In` a dependence of $T_{\mathtt{C}}$ on a disorder $\frac{1}{k_F l}$ has a maximum, but finally the strong disorder leads to decrease of $T_{\mathtt{C}}$ always [25], strong superconductors (`Pb`, `Hg`) have not this maximum [16–18]. In the experiments a total pattern was found: collapse of superconducting state takes place near Anderson's transition metal-insulator, that is when $\frac{1}{k_F l} \gtrsim 1$. It should be notice that superconduction appears in amorphus films of `Bi,Ga,Be` ($T_{\mathtt{C}} \sim 10\mathtt{K}$) just when these materials are not superconductors in a crystal state [30]. In such systems superconducting is result of intensification of electron-phonon interaction by disorder. Nowadays universal mechanisms of influence of a disorder upon $T_{\mathtt{C}}$ are unknown. Several mechanisms of degradation of $T_{\mathtt{C}}$ were supposed: a growth of Coulomb pseudopotential μ^* [19–21], influence of the disorder upon a density of states on Fermi surface $\nu(\xi)$ [22, 23] - evolution of Altshuler-Aronov singularity [3, 24] into "Coulomb gap". We will not consider these phenomenons as violation of Anderson's theorem because they have other nature and we will consider a weak disorder $\frac{1/l}{k_F} \ll 1$ that is far from a metal-insulator transition.

Introduction of nonmagnetic impurities in a superconductor is widely used in a practice: the impurities essentially increase a critical current and a magnetic field but do not change critical temperature at the same time. Our problem

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is to find such impurities which violates Anderson's theorem in the direction of essentiality increasing of the critical temperature $T_{\mathtt{C}}$. Obviously it is matter of nonmagnetic impurities in a three-dimensional superconductor with s-wave order parameter Δ. The impurities have to play a role of a catalyst of superconductivity in an originally low-temperature superconductor.

Nowadays a theory of disordered systems has been well developed for elastic scattering of conduction electrons by impurities [3, 4, 9, 24]. In a total case the scattering can be inelastic that is an impurity's potential depends on time $\upsilon(t)$. In this case to develop a perturbation theory (to unlink and to sum a diagram series) is impossible. In a section II it will be shown that in a special case of a *retarded* interaction with impurities the perturbation theory can be built and a theory of disordered systems can be generalized. In a section III it will be shown these impurities in a superconductor violates Anderson's theorem in the direction of increasing of $T_{\mathtt{C}}$. A change of the critical temperature depends on both impurities' parameters and electronic parameters of a metal matrix. At specific combinations of the parameters the critical temperature can essentially exceed critical temperature of a pure metal and has values up to room temperature.

II. RETARDED INTERACTION OF CONDUCTION ELECTRONS WITH IMPURITIES.

Let us consider an electron moving in a field created by N scatterers (impurities) which are placed in a random manner with concentration $\rho = \frac{N}{V}$. A random distribution of the impurities in a space corresponds to a distribution function: $P(\mathbf{R}_j) = V^{-N}$. Let a potential of an impurity is a function of coordinates and time: $\upsilon(\mathbf{r} - \mathbf{R}_j, t)$, where $\mathbf{R}_j$ is an impurity's coordinate $\mathbf{r}$ is an electron's coordinate. A total potential created by the impurities is:

$$V(\mathbf{r}, t) = \sum_{j=1}^{N} \upsilon(\mathbf{r} - \mathbf{R}_j, t) = \frac{1}{V} \sum_{\mathbf{q}} \sum_{j} \upsilon(\mathbf{q}, t) e^{i\mathbf{q}(\mathbf{r} - \mathbf{R}_j)}, \tag{1}$$

where $\upsilon(\mathbf{q}, t)$ is Fourier transform of the potential, $\upsilon(-\mathbf{q}, t) = \upsilon^*(\mathbf{q}, t)$. In most cases the potential can be considered as point, so that $\upsilon(\mathbf{q}) \approx \upsilon = \int \upsilon(\mathbf{r}) d\mathbf{r}$. Thus the system is spatially inhomogeneous and nonconservative.

Considering the potential as weak a perturbation theory can be constructed writing the secondary quantized interaction Hamiltonian of an electron with the field (1) as $H_{int} = \int d\mathbf{r} \psi^+(\mathbf{r}) V(\mathbf{r}, t) \psi(\mathbf{r})$. Then a perturbation theory series for an electron's propagator has a view:

$$\begin{aligned} G(1, 1') &= G_0(1, 1') + \int d2 G_0(1, 2) V(2) G_0(2, 1') \\ &+ \int d2 d3 G_0(1, 2) V(2) G_0(2, 3) V(3) G_0(3, 1') + \ldots, \end{aligned} \tag{2}$$

where $1 \equiv (\mathbf{r}, t), 1' \equiv (\mathbf{r}', t')$. The averaging over an ensemble of samples with all possible positions of impurities recovers spatial homogeneity of a system. In a representation of secondary quantization the averaging operation over a disorder has a view [34]:

$$G(x, x') = -i \frac{\left\langle \widehat{T} \psi^+(x) \psi(x') \widehat{U} \right\rangle_0}{\left\langle \widehat{U} \right\rangle_0} \longrightarrow \langle G(x, x') \rangle = -i \left\langle \frac{\left\langle \widehat{T} \psi^+(x) \psi(x') \widehat{U} \right\rangle_0}{\left\langle \widehat{U} \right\rangle_0} \right\rangle_{\mathtt{disorder}}, \tag{3}$$

where $\widehat{U}$ is an evolution operator, $\langle \ldots \rangle_0$ is done over a ground state of Fermi system and a lattice (in the numerator and the denominator separately). The averaging over the disorder is done in the following way - at first the propagator is calculated at the given disorder, and only then the averaging $\langle \ldots \rangle$ is done (the whole fraction is averaged). At averaging of a series (2) $G(\mathbf{r}, \mathbf{r}', t) \to \langle G(\mathbf{r}, \mathbf{r}', t) \rangle$ in a limit $\rho \to \infty, \upsilon^2 \to 0, \rho\upsilon^2 = \mathtt{const}$ the averages appear with factorized correlators:

$$\begin{aligned} \langle V(\mathbf{r}_1) V(\mathbf{r}_2) \rangle = \rho \upsilon^2 \delta(\mathbf{r}_1 - \mathbf{r}_2), \qquad \langle V(1) \rangle = 0, \qquad \langle V(1) V(2) V(3) \rangle = 0, \ldots \\ \langle V(1) V(2) V(3) V(4) \rangle = \langle V(1) V(2) \rangle + \langle V(3) V(4) \rangle + \langle V(1) V(4) \rangle + \langle V(2) V(3) \rangle, \end{aligned} \tag{4}$$

that corresponds to motion of an electron in Gauss random field with a white noise correlator. Then an electron's propagator is determined with a sum of diagrams shown in Fig.1 (a diagrammatic techniques of averaging over disorder [3]). In an analytic view we have (we use rules of diagrammatic techniques presented in [26]):

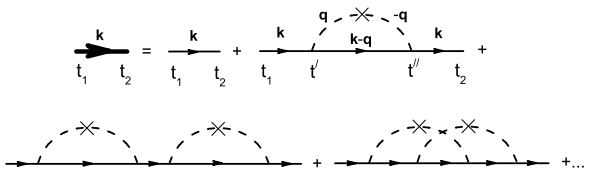

Figure 1: The diagram expansion of an averaged Green function $G(\mathbf{k},t)$ in a random field (4). Dotted lines with daggers means an action of an averaged summarized field of impurities in a momentum space - a transfer of an intermediate momentum $\mathbf{q}$.

$$iG(\mathbf{k},t_1,t_2) = \quad iG_0(\mathbf{k},t_2-t_1) + \int dt' \int dt'' iG_0(\mathbf{k},t'-t_1) iG_0(\mathbf{k},t_2-t'') \\ \cdot\rho \int \frac{d^3q}{(2\pi)^3}(-i)\upsilon(\mathbf{q},t') iG_0(\mathbf{k}-\mathbf{q},t''-t')(-i)\upsilon(-\mathbf{q},t'') + \ldots \tag{5}$$

Here $G_0(\mathbf{k},t_2-t_1)$ is a free electron's propagator depending on a time difference (a pure system is conservative) in Fermi system, G is a dressed electron's propagator. Since potential of an impurity is a function of a *point of time* $\upsilon = \upsilon(\mathbf{q},t)$, then diagrams of higher orders cannot be uncoupled, and the series (5) cannot be summed (energy is not conserved). The series can be summed partially in the following cases only. In the first case an impurity's potential does not depend on time $\upsilon = \upsilon(\mathbf{q})$. It means that an electron scatters elastically by impurities. It is well described by the disordered system theory [3, 4, 9]. Necessary to us concepts of the theory are presented in Appendix A.

In this article we propose another case when the the series (5) can be uncoupled and summed partially. The case is when an impurity's potential is a function of a time difference between consecutive scatterings. That is an interaction of electrons with impurities is retarded (advanced). In the first approximations a dependence of a scattering potential on a time difference can be considered as harmonic:

$$\upsilon(\mathbf{q},t')\upsilon(-\mathbf{q},t'') = \left\{ \begin{array}{ll} \upsilon(\mathbf{q})\upsilon(-\mathbf{q})e^{-i\omega_0(t''-t')}; & \texttt{for} \quad t''>t' \\ \upsilon(\mathbf{q})\upsilon(-\mathbf{q})e^{i\omega_0(t''-t')}; & \texttt{for} \quad t''<t' \end{array} \right\}, \tag{6}$$

This means that each impurity is a harmonic oscillator with an eigenfrequency ω_0. As a matter of convenience we can unite the retarded part and the advanced part in one expression:

$$\upsilon(\mathbf{q},t')\upsilon(-\mathbf{q},t'') = |\upsilon(\mathbf{q})|^2 \left[\theta_{t''-t'} e^{-i\omega_0(t''-t')} + \theta_{t'-t''} e^{i\omega_0(t''-t')}\right]$$

Let us substitute Fourier transforms of functions G_0, G and $\upsilon(\mathbf{q},t')\upsilon(-\mathbf{q},t'')$ into the series (5). Then we obtain:

$$\begin{aligned} iG(\mathbf{k},\varepsilon) &= iG_0(\mathbf{k},\varepsilon) + \left[iG_0(\mathbf{k},\varepsilon)\right]^2 \rho \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} iG_0(\mathbf{k}-\mathbf{q},\varepsilon-\omega)|\upsilon(\mathbf{q})|^2(-i)\frac{2\omega_0}{\omega^2-\omega_0^2+2i\delta\omega_0} + \ldots \\ &\equiv iG_0(\mathbf{k},\varepsilon) + \left[iG_0(\mathbf{k},\varepsilon)\right]^2 \rho \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} iG_0(\mathbf{k}-\mathbf{q},\varepsilon-\omega)|\upsilon(\mathbf{q})|^2(-i)D(\omega) + \ldots \end{aligned} \tag{7}$$

Thus we can see that *if impurities are harmonic oscillators with some eigenfrequency ω_0 then scattering of electrons by the impurities is equivalent to scattering of the electrons by "collective excitations" being described with a propagator and a coupling constant accordingly*

$$D(\omega) = \frac{2\omega_0}{\omega^2-\omega_0^2+2i\delta\omega_0}, \qquad \rho|\upsilon(\mathbf{q})|^2 \tag{8}$$

Since really in a system such "collective excitations" do not propagate then we will call the function $D(\omega)$ by *pseudo-propagator*. Thus *in consequence of the correlations $\langle V(1)V(2)\rangle \neq 0$ (4) we have a situation as though the "collective excitations" propagate through a system.* At increasing of impurities' density $\rho \sim N_0/V$ (N_0 is number of lattice sites of the matrix) the pseudopropagator does not pass to a phonon propagator in new lattice (in an obtained alloy) and the diagrams (Fig.1) - to electron-phonon interaction, because the pseudopropagator and the diagrams with daggers is consequence of above-mentioned correlations but phonon propagators in a solid is consequence of quasi-elastic interaction between atoms in a lattice. Moreover contributions of the diagrams (beginning from the second order) are proportional to $\upsilon^2\left(\rho-\rho^2\frac{V}{N_0}\right)$, $\upsilon^4\left(\rho-7\rho^2\frac{V}{N_0}+12\rho^3\left(\frac{V}{N_0}\right)^2-6\rho^4\left(\frac{V}{N_0}\right)^3\right)$ and so on. At large impurities' density

$\rho = N_0/V$ these contributions are zero. This means that theory of a disordered metal does not describe a transition to an alloy at increasing of impurities' density. In presented article we consider the density $\rho \ll \frac{N_0}{V}$.

High order corrections including any cross processes mentioned in Appendix A is selected in an analogous way in a series (5). We can sum the series with standard method and obtain Dyson equation $\frac{1}{G_0} = \frac{1}{G} - i\Sigma$ (in temperature technics already). A mass operator is written in a form:

$$\begin{aligned} -\Sigma(\mathbf{k},\varepsilon_n) = \quad & T\sum_{n'=-\infty}^{+\infty}\int \frac{d^3q}{(2\pi)^3}\rho|v(\mathbf{q})|^2 iG_0(\mathbf{k}-\mathbf{q},\varepsilon_n-\omega_{n'})iD(\omega_{n'}) \\ & +T^2\sum_{n'=-\infty}^{+\infty}\sum_{n''=-\infty}^{+\infty}\int \frac{d^3q}{(2\pi)^3}\int \frac{d^3p}{(2\pi)^3}\rho^2|v(\mathbf{q})|^2|v(\mathbf{p})|^2 iG_0(\mathbf{k}-\mathbf{q},\varepsilon_n-\omega_{n'}) \\ & iG_0(\mathbf{k}-\mathbf{q}-\mathbf{p},\varepsilon_n-\omega_{n'}-\omega_{n''})iG_0(\mathbf{k}-\mathbf{p},\varepsilon_n-\omega_{n''})iD(\omega_{n'})iD(\omega_{n''})+\ldots \end{aligned} \tag{9}$$

Corresponding diagrams are presented in Fig.2. In self-consistent theory the internal electron lines in the diagrams

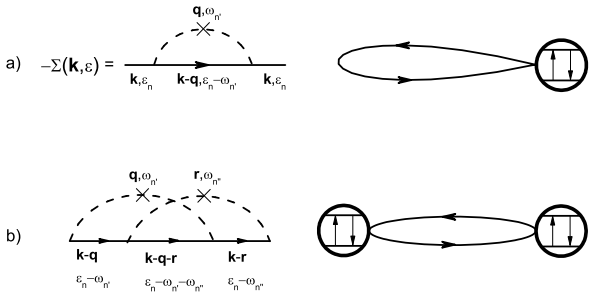

Figure 2: Mass operators describing a multiple scattering of electrons by impurities. These diagrams is analogous to the diagrams in Fig.7. However now the dotted lines with daggers mean a scattering by the impurities with energy transfer, and a multiplier $\rho v^2 iD(\omega_n)$ - pseudopropagator (8) is related to them. These processes can be interpreted with pictures on the right - the scattering induces transitions of the impurity-oscillator with eigenfrequency ω_0 between levels.

must be bold $G_0 \to G$. Since in metals $1/k_F \simeq a$ (a is lattice constant) then a weak disorder corresponds to $\frac{1}{k_F l} \ll 1$. A small parameter for the expansion (9) is a ratio of a contribution of cross diagrams to a contribution of diagrams without crossings. At inelastic scattering by impurities a particle's energy can change by a value $\triangle\varepsilon \sim \omega_0$, that corresponds to a momentum's uncertainty $\triangle k = \frac{m\omega_0}{k_F}$. In [3] was shown that the ratio is $\frac{\triangle k}{k_F}$, then

$$\frac{\triangle k}{k_F} = \frac{m\omega_0}{k_F^2} \sim \frac{\omega_0}{\varepsilon_F} \ll 1. \tag{10}$$

This expression likes a situation with phonons where an order of smallness is an adiabaticity parameter (Migdal's theorem). It is necessary to notice that the cross diagrams give a small contribution because, as it was noted in Appendix A, they describe an interference contribution in scattering by impurities, however processes of this type are strongly suppressed in consequence of an inelastic interaction with the impurities.

III. VIOLATION OF ANDERSON'S THEOREM.

A. Basic equations.

Let we have a metal with an attractive interaction between electrons: $\lambda - \mu^* > 0$ (λ is an electron-phonon coupling constant, $\mu^* > 0$ is Coulomb pseudopotential). Then the metal can be superconductor. Besides s-wave pairing takes a place (with zero orbital moment of a pair and zero summary spin). In the simplest case a superconductive gap is described with a self consistent equation (it is analogously for Δ):

$$\Delta^+(\varepsilon_n) = gT\sum_{n'=-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\xi(-i)F^+(\varepsilon_{n'},\xi)w_{\omega_D}(\varepsilon_n,\varepsilon_{n'}) \tag{11}$$

where $g \approx \lambda - \mu^*$ is an electron-electron coupling constant, $F(\varepsilon_n, \xi)$ is an anomalous propagator (propagator of a pair):

$$F(\varepsilon_n, \xi) = \frac{i\Delta(\varepsilon_n)}{(i\varepsilon_n)^2 - E^2}, \quad F^+(\varepsilon_n, \xi) = \frac{-i\Delta^+(\varepsilon_n)}{(i\varepsilon_n)^2 - E^2}, \tag{12}$$

$E^2 = \xi^2 + |\Delta(\varepsilon_n)|^2$ is a dispersion law of boholons. A function $w_{\omega_{\mathrm{D}}}(\varepsilon_n, \varepsilon_{n'})$ cuts the interaction because the pairing interaction is effective if energies of interacting quasi-particles are less than a characteristic frequency: $|\varepsilon_n| \leq \omega_D$. We can suppose the gap to be real $\Delta = \Delta^+$ and to depend on energy as follows [34]:

$$\Delta(\varepsilon_n) = \Delta \frac{\omega_D}{\sqrt{\varepsilon_n^2 + \omega_D^2}} \equiv \Delta w_{\omega_{\mathrm{D}}}(\varepsilon_n). \tag{13}$$

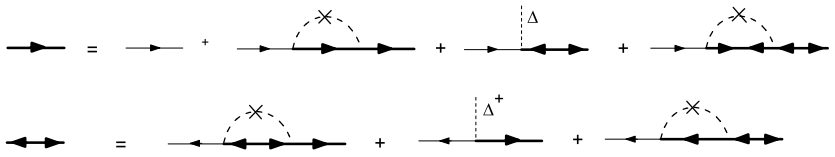

Figure 3: Gor'kov equations for a dirty superconductor. Interaction of quasiparticles with impurities is retarded. A pseudopropagator of excitations of the impurities $\rho v^2 iD(\omega_n)$ is associated with lines of a scattering. Unidirectional thin lines correspond to free propagators G_0 (A1) and G_0^+ (with a reverse arrow). Unidirectional bold lines correspond to dressed normal propagators $\widetilde{G}$ and $\widetilde{G}^+$ (15). Bold lines with oppositely directed arrows correspond to dressed anomalous propagators $\widetilde{F}$ and $\widetilde{F}^+$ (15).

Let us inject impurities into a above described metallic matrix. The impurities interact with conduction electrons of the metal in a retarding manner. In the simplest case the impurity is modelled with the oscillator (6) with eigenfrequency ω_0. As it is well known Gor'kov equation for a dirty superconductor has view shown in Fig.3. The diagrams are analogous to diagrams of a dirty superconductor with elastic impurities (Apendix A). Their sense is that electrons pair in the metallic matrix at first, then normal and anomalous propagators are dressed by interaction with impurities [33]. However now lines of the interaction (dotted lines) transfer energy. We neglect cross diagrams (the diagrams like in Fig.2b). For elastic scattering a small parameter of contribution of the cross diagrams is $\frac{1}{k_F l} \ll 1$ [3]. For a retarded interaction with impurities the small parameter is a ratio (10) $\frac{\omega_0}{\varepsilon_F} \ll 1$. Either as in [3] we suppose the order parameter is selfaveraging: $\langle \Delta^2(\mathbf{r}) \rangle - \langle \Delta(\mathbf{r}) \rangle^2 = 0$. This means to neglect a scattering of Cooper pairs by fluctuations of the gap, it is valid at $\frac{1}{k_F l} \ll 1$. In an analytic view the equations in Fig.3 are:

$$\left\{ \begin{array}{c} \widetilde{G}(i\widetilde{\varepsilon}_n - \xi) + \widetilde{F}^+ \widetilde{\Delta} = i \\ \\ \widetilde{F}^+(i\widetilde{\varepsilon}_n + \xi) + \widetilde{G}\widetilde{\Delta}^+ = 0 \end{array} \right\} \tag{14}$$

Solutions of the set of equations are normal and anomalous propagators:

$$\widetilde{G}(\varepsilon_n, \xi) = i \frac{i\widetilde{\varepsilon}_n + \xi}{(i\widetilde{\varepsilon}_n)^2 - \xi^2 - |\widetilde{\Delta}(\varepsilon_n)|^2}, \quad \widetilde{F}^+(\widetilde{\varepsilon}_n, \xi) = \frac{-i\widetilde{\Delta}^+(\varepsilon_n)}{(i\widetilde{\varepsilon}_n)^2 - \xi^2 - |\widetilde{\Delta}(\varepsilon_n)|^2}, \tag{15}$$

where a renormalized gap $\widetilde{\Delta}$ and a renormalized energetic parameter $\widetilde{\varepsilon}_n$ are determined with equations (here $\varepsilon_n = (2n+1)\pi T$ and $\varepsilon_{n'} = (2n'+1)\pi T$):

$$\begin{aligned} \widetilde{\Delta}^+(\varepsilon_n) &= \Delta^+(\varepsilon_n) + \rho \int \frac{d^3p}{(2\pi)^3} |v(\mathbf{k}-\mathbf{p})|^2 T \sum_{n'=-\infty}^{+\infty} \left(-i\widetilde{F}^+(\mathbf{p}, \varepsilon_{n'}) \right) iD(\varepsilon_n - \varepsilon_{n'}) \qquad (16) \\ &= \Delta^+(\varepsilon_n) + \rho |v|^2 \nu_F \frac{2}{\omega_0} \sum_{n'=-\infty}^{+\infty} \frac{\pi T \widetilde{\Delta}^+(\varepsilon_{n'})}{\sqrt{\widetilde{\varepsilon}_{n'}^2(\varepsilon_{n'}) + |\widetilde{\Delta}(\varepsilon_{n'})|^2}} \frac{\omega_0^2}{(\varepsilon_n - \varepsilon_{n'})^2 + \omega_0^2} \\ i\widetilde{\varepsilon}_n(\varepsilon_n) &= i\varepsilon_n + \rho \int \frac{d^3p}{(2\pi)^3} |v(\mathbf{k}-\mathbf{p})|^2 T \sum_{n'=-\infty}^{+\infty} i\widetilde{G}(\mathbf{p}, \varepsilon_{n'}) iD(\varepsilon_n - \varepsilon_{n'}) \qquad (17) \end{aligned}$$

$$= i\varepsilon_n + \rho\left|\upsilon\right|^2\nu_F\frac{2}{\omega_0}\sum_{n'=-\infty}^{+\infty}\frac{\pi T i\widetilde{\varepsilon}_{n'}(\varepsilon_n)}{\sqrt{\widetilde{\varepsilon}^2_{n'}(\varepsilon_{n'})+|\widetilde{\Delta}(\varepsilon_{n'})|^2}}\frac{\omega_0^2}{(\varepsilon_n-\varepsilon_{n'})^2+\omega_0^2}.$$

Since an interaction with an impurity is short-range then we can suppose $\upsilon(\mathbf{k}-\mathbf{p})\approx\upsilon=\texttt{const}$. Since spectrum of quasiparticles near Fermi surface is linear then integration over momentums can be simplified: $\frac{d^3p}{(2\pi)^3}\approx\nu_F d\xi$. The gap Δ is determined by the same equation (11) however with a dressed anomalous propagator $\widetilde{F}$ from Eq.(15):

$$\Delta^+(\varepsilon_n)=gT\sum_{n'=-\infty}^{+\infty}\int_{-\infty}^{+\infty}d\xi(-i)\widetilde{F}^+(\varepsilon_{n'},\xi)w_{\omega_{\mathtt{D}}}(\varepsilon_n,\varepsilon_{n'}) \tag{18}$$

Our problem is to calculate the parameters $\widetilde{\Delta}$ and $\widetilde{\varepsilon}_n$, and with them to calculate critical temperature $\Delta(T)=0$ in a system metal matrix+impurities. It should be noted that these function is determined self-consistently, that is any corrections to their values in a pure metal Δ,ε_n is determined by the sought quantities $\widetilde{\Delta},\widetilde{\varepsilon}_n$. In a general case to solve the set of equations (16,17,18) is very problematically. We will consider some limit cases and approximations.

B. A limit case of high temperature $T\gg\omega_0$.

In a case $T\gg\omega_0$ we can neglect an energy transfer along lines of interaction with an impurity because $(\varepsilon_n-\varepsilon_{n'})^2=4\pi^2T^2(n-n')^2\gg\omega_0^2$, if $n'\neq n$. That is we have

$$\frac{\omega_0^2}{(\varepsilon_n-\varepsilon_{n'})^2+\omega_0^2}\rightarrow\begin{array}{ll}\frac{\omega_0^2}{4\pi^2T^2(n-n')^2}\rightarrow 0 & \texttt{for}\quad n'\neq n\\ 1 & \texttt{for}\quad n'=n\end{array} \tag{19}$$

Thus in a sum $\sum_{n'=-\infty}^{+\infty}$ terms with $n'=n$ survive only. Then Eqs.(16,17) are reduced to

$$\begin{aligned}\widetilde{\Delta}^+&=\Delta^+ + \frac{2\pi\rho\left|\upsilon\right|^2\nu_F T}{\omega_0}\frac{\widetilde{\Delta}^+}{\sqrt{\widetilde{\varepsilon}^2_{n'}+|\widetilde{\Delta}|^2}}\\ i\widetilde{\varepsilon}_n&=i\varepsilon_n+\frac{2\pi\rho\left|\upsilon\right|^2\nu_F T}{\omega_0}\frac{i\widetilde{\varepsilon}_n}{\sqrt{\widetilde{\varepsilon}^2_{n'}+|\widetilde{\Delta}|^2}}.\end{aligned} \tag{20}$$

We can see that the limit $T\gg\omega_0$ corresponds to an elastic scattering by impurities with a scattering frequency $\frac{1}{2\pi\tau}=\rho\upsilon^2\nu_F\frac{2T}{\omega_0}$. Solving Eq.(20) we find that the gap and the energetic parameter are renormalized similarly:

$$\frac{\widetilde{\Delta}}{\Delta}=\frac{\widetilde{\varepsilon_n}}{\varepsilon_n}=1+\frac{1}{2\tau}\frac{1}{\sqrt{\varepsilon_n^2+\Delta^2}}. \tag{21}$$

The relation (21) means realization of Anderson's theorem (the gap and, accordingly, critical temperature do not change):

$$\Delta=g\pi T\sum_{n=-\infty}^{+\infty}\frac{\widetilde{\Delta}}{\sqrt{\widetilde{\varepsilon}^2_{n'}+\widetilde{\Delta}^2}}=g\pi T\sum_{n=-\infty}^{+\infty}\frac{\Delta}{\sqrt{\varepsilon^2_{n'}+\Delta^2}}. \tag{22}$$

Thus at temperatures being much more than oscillation frequency of impurities ω_0 the scattering by the impurities influences trivially upon supervonducting properties of a metal: it does not change a critical temperature and it reduces a coherence length $\frac{1}{\xi}=\frac{1}{\xi_0}+\frac{1}{l}$.

C. A self-consistent approximate solution for a general case.

From Eqs.(16,17) we can see that *in a general case the gap and the energetic parameter are renormalized differently due a retarded interaction with impurities. Hence Anderson's theorem is violated.* This is a result of the fact that

the equations include a summation over the energetic parameter (an index n') unlike an elastic case, and under the summation sign different functions are - $\widetilde{\Delta}$ in the equation (16) (even) and $\widetilde{\varepsilon}_n$ in the equation (17) (odd). The set of equations (16,17) can be simplified using an approximation of an electron-electron interaction amplitude $gw(\varepsilon_n, \varepsilon_{n'})$ with a method stated in [34]:

$$gw(\varepsilon_n, \varepsilon_{n'}) = g\frac{\omega^2}{(\varepsilon_n - \varepsilon_{n'})^2 + \omega^2} \longrightarrow gw(\varepsilon_n)w(\varepsilon_{n'}) = g\frac{\omega}{\sqrt{\varepsilon_n^2 + \omega^2}}\frac{\omega}{\sqrt{\varepsilon_{n'}^2 + \omega}}. \tag{23}$$

Here $\omega = \omega_{\mathtt{D}}, \omega_0, \ldots$ is characteristic frequency of the interaction. The approximation (23) corresponds to separation of a contribution of terms with $n' \neq n$ because at $T \gg \omega_0$ we have a limit:

$$\frac{\pi T}{\omega_0}\frac{\omega_0^2}{\varepsilon_n^2 + \omega_0^2} = \frac{\pi T}{\omega_0}\frac{(\omega_0/\pi T)^2}{(2n+1)^2 + (\omega_0/\pi T)^2} \longrightarrow \frac{\pi T}{\omega_0}\left(\frac{\omega_0}{\pi T}\right)^2\frac{1}{(2n+1)^2} \to 0$$

The terms with $n' = n$ in sums (16,17) neglected in the the approximation (23) do not influence upon a gap and a critical temperature. The terms describe a scattering of electrons by thermal oscillations of an impurity. The thermal oscillations behave like static impurities with effective concentration $\rho\frac{2T}{\omega_0}$. The scattering gives an additional contribution in a metallic resistance analogously to a contribution of thermal phonons. Thus terms with $n' \neq n$ can violate Anderson's theorem only.

Due the approximation (23) Eqs.(16,17) take forms:

$$\widetilde{\Delta}^+(\varepsilon_n) = \Delta^+(\varepsilon_n) + \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n'=-\infty}^{+\infty}\frac{\pi T\widetilde{\Delta}^+(\varepsilon_{n'})}{\sqrt{\widetilde{\varepsilon}_{n'}^2 + |\widetilde{\Delta}(\varepsilon_{n'})|^2}}w(\varepsilon_n)w(\varepsilon_{n'}) \tag{24}$$

$$\widetilde{\varepsilon}_n = \varepsilon_n + \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n'=-\infty}^{+\infty}\frac{\pi T\widetilde{\varepsilon}_{n'}}{\sqrt{\widetilde{\varepsilon}_{n'}^2 + |\widetilde{\Delta}(\varepsilon_{n'})|^2}}\frac{\omega_0}{\sqrt{\varepsilon_n^2 + \omega_0^2}}\frac{\omega_0}{\sqrt{\varepsilon_{n'}^2 + \omega_0}} = \varepsilon_n + 0. \tag{25}$$

In Eq.(25) in the second term under the summation sign a odd function of n' is, hence the energetic parameter is not renormalized $\widetilde{\varepsilon}_n = \varepsilon_n$. Let us consider a case when Debye frequency of the matrix and a frequency of impurities coincide: $\omega_{\mathtt{D}} = \omega_0$. A dependence of a gap on energy we can write in a view $\Delta(\varepsilon_n) = \Delta w_{\omega_{\mathtt{D}}}(\varepsilon_n) = \Delta w_{\omega_0}(\varepsilon_n)$ and $\widetilde{\Delta}(\varepsilon_n) = \Delta w_{\omega_0}(\varepsilon_n)$ (as in Eq.(13)). Then Eq.(24) is simplified:

$$\widetilde{\Delta}^+ = \Delta^+ + \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n'=-\infty}^{+\infty}\frac{\pi T\widetilde{\Delta}^+}{\sqrt{\varepsilon_{n'}^2 + |\widetilde{\Delta}|^2 w^2(\varepsilon_{n'})}}w^2(\varepsilon_{n'}) \tag{26}$$

The equation can be rewritten in a view:

$$\widetilde{\Delta}^+ = \frac{\Delta^+}{1 - \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{(2n+1)^2 + \left(\widetilde{\Delta}/\pi T\right)^2 w^2(n)}}\frac{(\omega_0/\pi T)^2}{(2n+1)^2 + (\omega_0/\pi T)^2}} \tag{27}$$

The equation must be solved for $\widetilde{\Delta}$. Obtained solution has to be substituted into Eq.(18) determining the gap $\Delta(T)$ in a system matrix+impurities. In a case $\omega_{\mathtt{D}} \neq \omega_0$ Eq.(24) can be reduced to a view:

$$\widetilde{\Delta}^+(\varepsilon_n) = \Delta^+ w_{\omega_{\mathtt{D}}}(\varepsilon_n) + \Delta^+ w_{\omega_0}(\varepsilon_n)\frac{f}{1-h}, \tag{28}$$

where functions f and h are

$$\begin{aligned} f &= \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{(2n+1)^2 + \left(\widetilde{\Delta}(\varepsilon_n)/\pi T\right)^2}}\frac{(\omega_{\mathtt{D}}/\pi T)}{\sqrt{(2n+1)^2 + (\omega_{\mathtt{D}}/\pi T)^2}}\frac{(\omega_0/\pi T)}{\sqrt{(2n+1)^2 + (\omega_0/\pi T)^2}} \\ h &= \rho\left|v\right|^2\nu_F\frac{2}{\omega_0}\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{(2n+1)^2 + \left(\widetilde{\Delta}(\varepsilon_n)/\pi T\right)^2}}\frac{(\omega_0/\pi T)^2}{(2n+1)^2 + (\omega_0/\pi T)^2} \end{aligned} \tag{29}$$

If $\omega_{\mathtt{D}} = \omega_0$ we have $h = f$, hence $\widetilde{\Delta}^+(\varepsilon_n) = \widetilde{\Delta}^+ w_{\omega_0} = \Delta^+ w_{\omega_0}(\varepsilon_n)\frac{1}{1-h}$ that coincides with a previously obtained equation (27).

1. Critical temperature.

The problem is essentially simplified if we find critical temperature only. Then $\widetilde{\Delta}(T_{\mathtt{C}}) = \Delta(T_{\mathtt{C}}) = 0$ and Eq.(28) is reduced to a view:

$$\widetilde{\Delta}^{+}(\varepsilon_n) = \Delta^{+} w_{\omega_{\mathtt{D}}}(\varepsilon_n) + \Delta^{+} w_{\omega_0}(\varepsilon_n) \frac{\frac{2\rho|v|^2\nu_F}{\pi T} \Upsilon\left(\frac{\omega_{\mathtt{D}}}{\pi T}, \frac{\omega_0}{\pi T}\right)}{1 - \frac{2\rho|v|^2\nu_F}{\pi T} \Xi\left(\frac{\omega_0}{\pi T}\right)}, \tag{30}$$

where Ξ is a function which we will name an *effectiveness function* (Fig.4):

$$\Xi\left(\frac{\omega_0}{\pi T}\right) = \frac{\pi T}{\omega_0}\left[\gamma + 2\ln 2 + \frac{1}{2}\Psi\left(\frac{1}{2} - \frac{i}{2}\frac{\omega_0}{\pi T}\right) + \frac{1}{2}\Psi\left(\frac{1}{2} + \frac{i}{2}\frac{\omega_0}{\pi T}\right)\right]. \tag{31}$$

Here Ψ is a digamma function , $\gamma \approx 0.577$ is Euler constant. *The effectiveness function describes an influence of the impurities upon a superconductor depending on their oscillation frequency* ω_0. *The effectiveness function* $\Xi\left(\omega_0, T^*_{\mathtt{C}}\right)$ *determines such an oscillation frequency of an impurity to get the critical temperature* $T^*_{\mathtt{C}}$ *with the least concentration of the impurities.* On the one hand the smaller the frequency the stronger a scattering of electrons by the impurities $\rho\left|v\right|^2\nu_F\frac{2}{\omega_0}$, because it is necessary less expenditure of electron's energy to "swing" an oscillator. On the other hand at temperatures $T \gtrsim \omega_0$ a thermal noise destroys changes of oscillators' states by electrons. Thus in a region of frequencies and temperatures $T \gg \omega_0$ the impurities' effectiveness falls $\Xi\left(\frac{\omega_0}{T} \to 0\right) \to \frac{7}{4}\zeta(3)\frac{\omega_0}{\pi T} \to 0$. This result is in agreement with a result in Subsection III B. At $\omega_0 \gg T$ the oscillations are "freezed" and energy level transitions are determined by an interaction with metal's electrons only. However for too large frequencies $\omega \sim \varepsilon_F$ the interaction is weak. Hence in a region of large frequencies ω_0 the effectiveness is slowly decreasing: $\Xi\left(\frac{\omega_0}{T} \to \infty\right) \to \frac{\pi T}{\omega_0}\ln\left(\frac{2}{\gamma}\frac{\omega_0}{\pi T}\right) \to 0$. An optimal value of the oscillation frequency is $\frac{\omega_0}{\pi T} = 1.09$ at given temperature when the effectiveness function reaches its maximum value $\Xi_{\mathtt{max}} = 1.10$ (Fig.4). A function Υ is analogous to the effectiveness function Ξ with the difference that it depends on both a matrix's frequency $\omega_{\mathtt{D}}$ and an impurity's frequency ω_0. However it does not play a principal role because it is in the numerator. If the frequencies are equal $\omega_{\mathtt{D}} = \omega_0$ then Eq.(30) passes into Eq.(??).

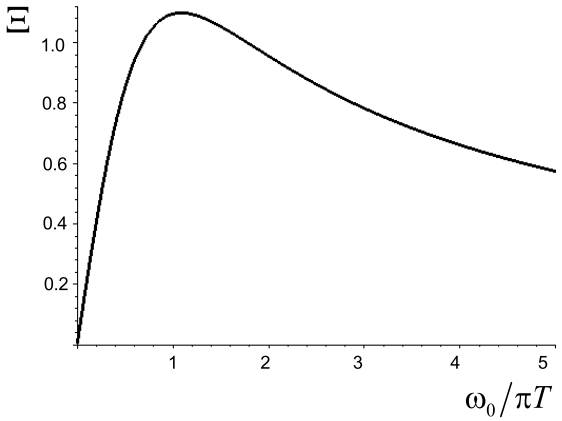

Figure 4: The effectiveness function $\Xi\left(\omega_0/T\right)$ as a function of the ratio of an impurity's oscillation frequency to temperature.

Now let us consider a multiplier $\frac{2\rho|v|^2\nu_F}{\pi T}$ in a formula (30). By analogy with a scattering by elastic impurities (Appendix A) the value $\frac{1}{\tau} = 2\pi\rho v^2\nu_F$ can be considered as a scattering frequency. Then a free length equals:

$$l = v_F\tau = \frac{v_F}{2\pi\rho v^2\nu_F} = \frac{k_F}{2\pi\rho v^2\nu_F m} \tag{32}$$

then

$$\frac{2\rho\left|v\right|^2\nu_F}{\pi T} \equiv \frac{1/\tau}{\pi^2 T} = \frac{2\varepsilon_F}{\pi^2 T}\frac{1}{k_F l}. \tag{33}$$

It should be noted that real values of a ratio of a reverse free length $1/l$ to Fermi momentum of a matrix k_F (which is equal to reverse interatomic distance $k_F \sim 1/a$) is $\frac{1}{k_F l} \sim \frac{a}{l} \ll 1$. When $\frac{1}{k_F l} \gtrsim 1$ a transition in a state of Anderson insulator can take a place [3, 4] (a localization with impurities). However the scattering being essentially inelastic, the transition can be suppressed.

Substituting the renormalized gap (30) $\widetilde{\Delta}$ in Eq.(18) and using the approximations (13) and (23) we obtain an equation to find critical temperature:

$$\Delta^{+}(\varepsilon_n) = gT \sum_{n'=-\infty}^{+\infty} \frac{\pi \widetilde{\Delta}^{+}(\varepsilon_n)}{|\varepsilon_{n'}|} w_{\omega_{\mathtt{D}}}(\varepsilon_n, \varepsilon_{n'})$$

$$\Rightarrow 1 = g \sum_{n'=-\infty}^{+\infty} \frac{\pi T}{|\varepsilon_{n'}|} \left[w^2_{\omega_{\mathtt{D}}}(\varepsilon_{n'}) + w_{\omega_{\mathtt{D}}}(\varepsilon_{n'}) w_{\omega_0}(\varepsilon_{n'}) \frac{\frac{2\varepsilon_F}{\pi^2 T} \frac{1}{k_F l} \Upsilon\left(\frac{\omega_{\mathtt{D}}}{\pi T}, \frac{\omega_0}{\pi T}\right)}{1 - \frac{2\varepsilon_F}{\pi^2 T} \frac{1}{k_F l} \Xi\left(\frac{\omega_0}{\pi T}\right)} \right] \tag{34}$$

In a limit $l \to \infty$ the equation (34) pass into equation (11) for a pure superconductor. Graphicly Eq.(34) is shown in Fig.5. The critical temperature is determined by an intersection of the line 1 and the second term of Eq.(34) as a function of temperature. The curve (a) determines critical temperature of a pure superconductor $T_{\mathtt{C}}$. The more a coupling constant g the more $T_{\mathtt{C}}$. The curve (b) determines critical temperature $T^*_{\mathtt{C}}$ of a system metal+impurities. The temperature T^* is determined by a zero in denominators in the formula (34):

$$\frac{2\varepsilon_F}{\pi^2 T^*} \frac{1}{k_F l} \Xi\left(\frac{\omega_0}{\pi T^*}\right) = 1 \Longleftrightarrow \frac{1/\tau}{\pi^2 T^*} \Xi\left(\frac{\omega_0}{\pi T^*}\right) = 1. \tag{35}$$

Moreover we can see that an inequality $T^*_{\mathtt{C}} \gtrsim T^* > T_{\mathtt{C}}$ is valid. In a point $T = T^*$ the second member of Eq.(34) is singular $g \cdot \infty$. *The singularity appears due a combined consistent pairing action of matrix's phonons and impurities' oscillations on electrons on the assumption of the averaging over a disorder* (3), *with a correlator "white noise"* (4). *The intensification of the pairing has sense in the presence of electron-electron attraction in a matrix only* $g = \lambda - \mu^* > 0$. *Thus the impurities play a role of a catalyst of superconductivity.* The singularity temperature T^* is determined by electronic parameters of a matrix and a coupling constant with impurities $\rho \upsilon^2 \nu_F$. However T^* does not depend on a frequency of a pairing interaction in a matrix ω_D if only it is nonzero and the coupling constant g if only it corresponds to attraction $g = \lambda - \mu^* > 0$. Moreover it is necessary to notice that $T^*_{\mathtt{C}}$ differs from T^* little in consequence of a dependence of an interaction with impurities on temperature $\frac{1}{T^*} \Xi\left(\frac{\omega_0}{\pi T^*}\right)$. Therefore the temperature T^* can be used as a lower estimate of the critical temperature. Pure superconductors have the singularity temperature too, however it equals to zero always $T^*_{l=\infty} = 0$ (Fig.5).

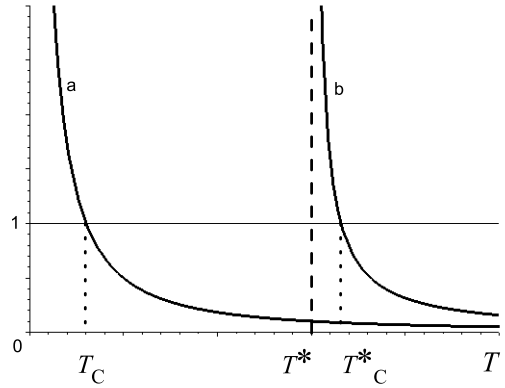

Figure 5: A graphic representation of the equations for the critical temperature. The curve (a) corresponds to Eq.(11) and determines the critical temperature of a pure superconductor $T_{\mathtt{C}}$. The curve (b) corresponds to Eq.(34) and determines critical temperature $T^*_{\mathtt{C}}$ of a system metal+impurities. In a point $T = 0$ the equation for a pure metal has a singularity. For the system metal+impureties the singularity exists at nonzero temperature T^*.

Estimation of the critical temperature $T^*_{\mathtt{C}}$ *states that it can essentially exceed critical temperature of a corresponding pure metal on the assumption of optimal choice of parameters of the matrix and the impurities.* For example, a pure

crystal of Al has critical temperature $T_C = 1.2\mathrm{K}$ corresponding to parameters $g = 0.17$, $\omega_D = 375\mathrm{K}$. Fermi energy and velocity are equal to $\varepsilon_F = 13.6 \cdot 10^4\mathrm{K}$, $v_F = 2.03 \cdot 10^6\mathrm{m/s}$ accordingly. Let the oscillation frequency is chosen in the optimal ratio to a desired temperature $\frac{\omega_0}{\pi T^*} \approx 1$, that is a value of the effectiveness function is $\Xi = 1 \approx \Xi_{max}$. Then we can plot the singularity temperature T^* as a function of a parameter $\frac{1}{k_F l}$ - Fig.6. In the figure we can see that if the parameter is $\frac{1}{k_F l} \approx 0.01 \ll 1$ the singularity temperature reaches giant (room) values $T^* \sim 300\mathrm{K}$ in comparison with critical temperature of the pure metal. It should be noted that in Fig.(6) *various values of l and, accordingly, various values of T^* correspond to various impurities chosen so that the frequency optimally measures against the temperature* $\frac{\omega_0}{\pi T^*} \approx 1$. With help an expression (33) we can calculate that to reach the critical temperature $\sim 300\mathrm{K}$ the free length can be $l \approx 12a$ where lattice constant of Al is $a = 4.08\mathrm{A}$.

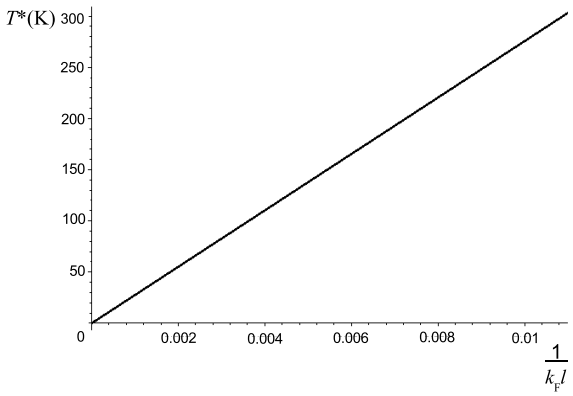

Figure 6: The singularity temperature T^* as a function of a parameter $\frac{1}{k_F l}$ for a matrix Al on the assumption that the oscillation frequency of impurities is chosen in an optimal ratio to the temperature $\frac{\omega_0}{\pi T^*} \approx 1$.

a. The gap. To solve the self-consistent equations (27) or (28)) for the gap $\widetilde{\Delta}$ at temperature $T < T_C^*$ is a more difficult problem than the previous one for critical temperature. In Eq.(34) for the critical temperature we have the singularity at temperature $T^* < T_C^*$ determined by Eq.(35) (zero in the denominator). However at presence of a nonzero gap $\widetilde{\Delta}$ the singularity is absent in consequence of self-consistency of the equations. Hence at any temperature $T < T_C^*$ the order parameter $\Delta(T)$ determined by Eq. (18) for a system matrix+impurities is a finite value.

IV. DISCUSSION.

In this article a theory of disordered metals is generalized if a retarded interaction of conduct electrons with impurities takes a place. In consequence of an averaging over the disorder we have diagram rules to be analogous to diagram rules for scattering by elastic impurities, however the lines of the interaction carry both momentum and energy. In a basic approximation an impurity is a harmonic oscillator with some frequency ω_0. As a result of correlations between successive scatterings we have a pattern that as though "collective excitations" propagates over the system, and scattering of metal's quasi-particles by impurities is determined by some coupling constant with the excitations depending on concentration of the impurities. As a result of inelasticity of the scattering by impurities the quantum contribution to conductivity (localization) can be suppressed. If temperature is much greater than the oscillation frequency of impurities $T/\omega_0 \gg 1$ then scattering by the impurities can be considered as elastic scattering by impurities with effective concentration $\rho\frac{2T}{\omega_0}$ (if only $\omega_0 \neq 0$).

Introduction of the impurities into three-dimension s-wave superconductor essentially influences on its superconductive properties. A gap and an energetics parameter are renormalizated differently due the retarded interaction of metal's quasi-particles with the impurities. This results in violation of Anderson's theorem in the direction of increasing of critical temperature. Influence of impurities upon the critical temperature determined by a reverse free length $1/l$ or a scattering frequency $1/\tau = v_F/l$. The critical temperature essentially depends on the oscillation frequency ω_0 of the impurities too. The dependence is described by an effectiveness function $\Xi\left(\omega_0, T_C^*\right)$. The function has a maximum determining the frequency ω_0 to obtain the critical temperature T_C^* with minimal concentration of the impurities: $\omega_0 = \pi T_C^*$. In limit cases $T \gg \omega_0$ and $\omega_0 \to \infty$ effectiveness of the impurities aspires to zero, because at too small frequency a thermal noise destroys the changes of oscillators' states by electrons, and at too large frequency

an interaction with the impurities is weak. Increase of the critical temperature is a result of a combined consistent action of metal's phonons and impurities' oscillation upon electrons under the condition of averaging over a disorder, where electrons move in Gauss random field with a white noise correlator. The extension of the pairing takes place at presence of initially attractive interaction between electrons in a matrix only. Thus the impurities play a role of catalyst of superconductivity.

Estimation of $T_{\mathtt{C}}^*$ shows that the critical temperature can essentially exceed critical temperature of the pure metal under the condition of optimal choice of parameters of the matrix and the impurities. So for a matrix from $\mathtt{Al}$ at the parameter value $\frac{1}{k_F l} \approx 0.01 \ll 1$ the critical temperature reaches giant (room) values $T_{\mathtt{C}}^* \sim 300\mathtt{K}$ in comparison with critical temperature of the pure metal $T_{\mathtt{C}} = 1.2\mathtt{K}$ under the condition of optimal relation between the critical temperature and an oscillation frequency of impurities $\omega_0 = \pi T_{\mathtt{C}}^*$. Thus the proposed model of the catalysis by impurities with retarded interaction gives a principal possibility to obtain high critical temperature at reasonable concentration of the impurities.

Appendix A: Elastic scattering by impurities

In a case of elastic scattering of electrons by impurities a impurity's potential is a function of a wave vector only $\upsilon = \upsilon(\mathbf{q})$. An electron propagator is a function $G_0(\mathbf{k}, t_2 - t_1)$ and its Fourier-transform is

$$G_0(\mathbf{k}, \varepsilon) = \int_{-\infty}^{+\infty} d(t_2 - t_1) e^{i\varepsilon(t_2 - t_1)} G_0(\mathbf{k}, t_2 - t_1) = \frac{1}{\varepsilon - \xi(k) + i\delta \mathtt{sign}\varepsilon}, \tag{A1}$$

where $\xi(k) = \frac{k^2}{2m} - \varepsilon_F \approx v_F(k - k_F)$ is energy of an electron counted from Fermi surface, ε is an energy parameter, the series (5) can be represented in a view of Dyson equation:

$$iG(\mathbf{k}, \varepsilon) = iG_0(\mathbf{k}, \varepsilon) + iG_0(\mathbf{k}, \varepsilon)(-i)\Sigma iG(\mathbf{k}, \varepsilon), \tag{A2}$$

where $\Sigma(\mathbf{k}, \varepsilon)$ is a mass operator. For a weak disorder $\frac{1}{k_F l} \ll 1$ (l is a free length) it is possible to neglect the cross diagrams and to write the mass operator in a view (Fig.7):

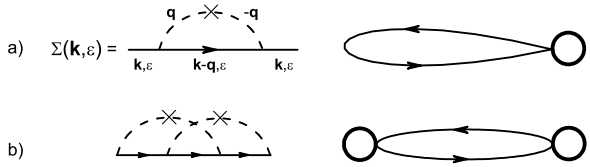

Figure 7: Mass operators describing a multiple scattering of electrons by impurities. Dotted lines with daggers on the diagrams mean the scattering without an energy transfer, and a multiplier $\rho\upsilon^2$ is related to them. A diagram (a) describes the second Born approximation with an amplitude of a scattering potential υ. The diagram can be interpreted by the picture on the right - an infinite to one scattering by an impurity. A cross diagram (b) describes a quantum correction to the scattering - interference of incident and reflected by impurities electron waves. This diagram can be interpreted as an infinite to one scattering by two impurities with superposition of the scattered waves.

$$-\Sigma(\mathbf{k}, \varepsilon_n) = \rho \int \frac{d^3q}{(2\pi)^3} (-1)\upsilon(\mathbf{q}) iG_0(\mathbf{k} - \mathbf{q}, \varepsilon_n)(-1)\upsilon(-\mathbf{q}) = \rho \int \frac{d^3p}{(2\pi)^3} |\upsilon(\mathbf{k} - \mathbf{p})|^2 iG_0(\mathbf{p}, \varepsilon_n), \tag{A3}$$

where we passed to Matsubara representation (nonzero temperature $\varepsilon_n = (2n + 1)\pi T$). it should be noted that in the diagrams the dotted lines are not dressed with polarization loops, because the disorder is "freezed in" and the impurities do not fit into changes of an electron density. Substituting a free propagator $G_0(\mathbf{p}, \varepsilon_n) = \frac{i}{i\varepsilon_n - \xi(p)}$ into the expression for a mass operator we obtaining (supposing a weak dependence of a impurity's potential on momentum $\upsilon(\mathbf{k} - \mathbf{p}) \approx \upsilon$ and linear specter of quasi-particles near Fermi surface $\xi(k) \approx v_F(k - k_F)$):

$$\Sigma(\mathbf{p}, \varepsilon_n) = -i\frac{\varepsilon_n}{|\varepsilon_n|}\pi\rho\upsilon^2\nu_F \equiv -i\gamma \mathtt{sign}\varepsilon_n$$
$$G(\mathbf{k}, \varepsilon_n) = \frac{1}{G_0^{-1} + i\Sigma} = \frac{i}{i\varepsilon_n - \xi(p) + i\gamma \mathtt{sign}\varepsilon_n} \tag{A4}$$

where $\nu_F = \frac{mk_F}{2\pi^2}$ is a density of states on Fermi surface per one projection of spin. Then the mean free time and the free length are determined as [3]:

$$\tau = \frac{1}{2\gamma}, \qquad l = v_F\tau = \frac{v_F}{2\gamma} = \frac{v_F}{2\pi\rho v^2\nu_F} \tag{A5}$$

Elastic impurities do not influence upon effective mass of quasi-particles but they stipulate for a quasi-particles' damping $\gamma \mathtt{sign}\varepsilon_n$.

A small parameter for the perturbation theory is a ratio of a contributions of cross diagrams to to a contribution of diagrams without crossings [3]. Due scattering a momentum of an electron obtains an uncertainty $\triangle k \sim 1/l$. Then the small parameter is $\frac{\triangle k}{k_F} \sim \frac{1}{k_F l}$. Since in metals $1/k_F \simeq a$ (where a is a lattice constant) then a weak disorder corresponds to $\frac{1}{k_F l} \ll 1$. All diagrams with the crossing describe quantum corrections for conductivity - interference of incident and reflected by impurities electron waves. This leads to Anderson's localization - transition of a metal to an insulator state [3, 4, 9] when $\frac{1}{k_F l} \gtrsim 1$ (electrons are "blocked" between the impurities). However with increase of temperature (or if the system is in an external alternating field [27]) processes of a nonelastic scattering begin to play a role (electron-phonon processes - [28], electron-electron processes - [29]). The processes limit the coherence time of electron waves $\tau_\varphi < \infty$ (or the coherence length $L_\varphi < \infty$). If $\tau_\varphi < \tau$ (or $L_\varphi < l$) then the interference contribution is essentially suppressed (it takes place the phase failure) [34], and the cross diagram can be neglected.

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